\( a) \) Rotations

A transformation that leaves the scalar product invariant:

\[ x \cdot y = \sum_{i=1}^{n} x_i y_i \]

\[ x_i \rightarrow \theta x_i \quad x_i \rightarrow \theta x_i \]

invariant if \( \sum \theta_i x_i y_i = \delta x_i y_i \)

or \( \theta^T \theta = 1 \)

\( \theta^T \theta = 1 = \det \theta^T \det \theta = 1 \)

\( \det \theta = \pm 1 \)

\( \det \theta = -1 \): Reflections

\( b) \) Rotations in the plane

We can either rotate a vector by \( \theta \) or the coordinate system by \(-\theta\).

Generally, we will think of transforming the coordinate system.