$$\Rightarrow \quad \ddot{x} + \omega_0^2 (1 + h\cos(2\omega_0 + \varepsilon)t)\, x = 0$$

Look for solutions of the form

$$x = a(t)\cos(\omega_0 + \tfrac{1}{2}\varepsilon)t \; + b(t)\sin(\omega_0 + \tfrac{1}{2}\varepsilon)t$$

we assume that $\dot{a}(t) \sim \varepsilon\, a(t)$

$\dot{b}(t) \sim \varepsilon\, b(t)$

$$\Rightarrow \left.\begin{array}{l} \ddot{a} = \varepsilon^2 a(t) \\ \ddot{b} = \varepsilon^2 b(t) \end{array}\right\} \text{neglect}$$

$$\Rightarrow -\dot{a}(\omega_0 + \tfrac{1}{2}\varepsilon)\sin(\omega_0 + \tfrac{1}{2}\varepsilon)t$$
$$+ \dot{b}(\omega_0 + \tfrac{1}{2}\varepsilon)\cos(\omega_0 + \tfrac{1}{2}\varepsilon)t$$
$$-(\omega_0 + \tfrac{1}{2}\varepsilon)^2 (a(t)\cos(\omega_0 + \tfrac{\varepsilon}{2})t + b(t)\sin(\omega_0 + \tfrac{1}{2}\varepsilon)t)$$
$$+ \omega_0^2 (a(t)\cos(\omega_0 + \tfrac{1}{2}\varepsilon)t + b(t)\sin(\omega_0 + \tfrac{1}{2}\varepsilon)t)$$
$$+ h\omega_0^2\cos(2\omega_0 + \varepsilon)t\,(a(t)\cos(\omega_0 t + \tfrac{\varepsilon}{2}t) + b(t)\sin(\omega_0 + \tfrac{1}{2}\varepsilon)t)$$
$$= 0$$

2nd order

higher frequencies are treated as higher order

lowest order $\frac{1}{2}\cos(\omega_0 + \frac{\varepsilon}{2})t$ $\quad -\frac{1}{2}\sin(\omega_0 + \frac{1}{2})t$

coefficients of $\cos(\omega_0 + \frac{1}{2}\varepsilon)t$ and $\sin(\omega_0 + \frac{1}{2}\varepsilon)t$ should both vanish

$$\Rightarrow -\dot{a}(\omega_0 + \cancel{\tfrac{\varepsilon}{2}}) - \omega_0\varepsilon\, b - \frac{h}{2}\omega_0^2 b = 0$$
$$\dot{b}(\omega_0 + \cancel{\tfrac{\varepsilon}{2}}) - \omega_0\varepsilon\, a + \frac{h}{2}\omega_0^2 a = 0$$

look for solution of the form $a = a_0 e^{st}$, $b = b_0 e^{st}$

$$\Rightarrow \begin{pmatrix} -s(\omega_0) & -\omega_0\varepsilon - \frac{h}{2}\omega_0^2 \\ -\omega_0\varepsilon + \frac{h}{2}\omega_0^2 & s\,\omega_0 \end{pmatrix} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = 0$$