$$\Rightarrow \quad x_1(t) = \mu_1^{t/T}\,\Pi_1(t) \qquad x_2(t) = \mu_2^{\frac{t}{T}}\,\Pi_2(t)$$

purely periodic

Since $\frac{d}{dt}\left(\dot{x}_1 x_2 - \dot{x}_2 x_1\right) = 0$

$$\Rightarrow \quad \dot{x}_1 x_2 - \dot{x}_2 x_1 = \text{constant}$$

$$\left(\tfrac{1}{T}\log\mu_1\,\mu_1^{t/T}\Pi_1 + \mu_1^{t/T}\dot{\Pi}_1\right)\mu_2^{t/T}\Pi_2 - \left(\tfrac{1}{T}\log\mu_2\,\mu_2^{t/T}\Pi_2 + \mu_2^{t/T}\dot{\Pi}_2\right)\mu_1^{t/T}\Pi_1$$

$\Rightarrow$ it changes by $\mu_1\mu_2$ for $t \to t+T$

$$\Rightarrow \quad \mu_1\mu_2 = 1$$

coefficients of diff. eq. are real

$\Rightarrow$ complex conjugate of a solution is also a solution

$\Rightarrow$ $\mu_1 = \mu_2^*$ or $\mu_1 \in \mathbb{R}$ and $\mu_2 \in \mathbb{R}$

$\Rightarrow |\mu_1| = |\mu_2| = 1$ $\qquad\Rightarrow$ one of the solutions is unstable. This is called a parametric resonance.

We now consider the case

$$\omega^2(t) = \omega_0^2\,(1 + h\cos\gamma t) \qquad h \ll 1, \quad h > 0$$

We expand $\gamma = 2\omega_0 + \varepsilon$

(we will find that the resonance is strongest for $\varepsilon = 0$)