Vd Parametric Resonance

We consider a ho with a time dependent mass and spring constant

$$\frac{d}{dt}(m\dot{x}) + \kappa x = 0$$

$$\Rightarrow \dot{m}\dot{x} + m\ddot{x} + \kappa x = 0$$

define $\tau = \int^t \frac{dt}{m(t)}$

$$\Rightarrow \frac{dx}{d\tau} = \frac{dx}{dt} m(t)$$

$$\frac{d^2x}{d\tau^2} = \frac{d^2x}{dt^2} m^2(t) + \frac{dx}{dt} \dot{m}\, m(t)$$

$$\Rightarrow \frac{d^2x}{d\tau^2} + m\kappa x = 0$$

This means that the most general problem is given by $\frac{d^2x}{dt^2} + \omega^2(t) x = 0$

assumption: $\omega(t)$ is periodic with period T

$$\omega(t+T) = \omega(t)$$

Solution

$$\Rightarrow \begin{pmatrix} x_1(t+T) \\ x_2(t+T) \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

we can always choose a basis where this matrix is diagonal

$$x_1(t+T) = \mu_1 x_1(t)$$

$$x_2(t+T) = \mu_2 x_2(t)$$