\[ \Theta(r=\infty) < \frac{\pi}{2} \text{ for attractive scatterer} \]
\[ \Rightarrow \Theta < 0 \]

repulsive scatterer

attractive scatterer

We now calculate \( \Theta \) for \( V(r) = -\frac{a}{r} \)

\[
-\Theta = -\pi + 2 \sqrt{b} \int_0^\infty \frac{dr}{r \sqrt{kr^2 + a^2}} - b^2
\]

\[
= -\pi + 2b \int_0^\infty \frac{du}{\sqrt{u + 1} - b^2 u}
\]

\[
= -\pi + 2b \int u_0 \sqrt{u + 1} - b^2 u du
\]

\[
u_0 = -\frac{2}{E + \sqrt{\frac{a^2}{E^2} + 4b^2}} \quad u = \frac{-a}{E} - \sqrt{\frac{a^2}{E^2} + 4b^2}
\]

Solve integral

\[
-\Theta = -\pi + 2 \cos^{-1} \left( \frac{u_0 - u_1}{u_0 - u_1} \right) - 2 \cos^{-1} \left( \frac{-u_0 + u_1}{u_0 - u_1} \right)
\]

\[
= -\pi + 2\pi - 2 \cos^{-1} \frac{\alpha/E}{\sqrt{\alpha/E + 4b^2}}
\]

\[
= -1 \cos \left( \frac{\pi}{2} + \frac{\Theta}{2} \right) = \frac{\alpha/E}{\sqrt{\alpha/E + 4b^2}}
\]