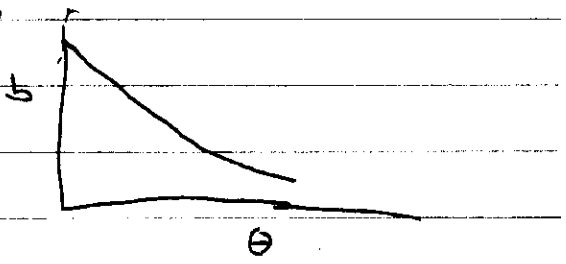
$$-J\, 2\pi\, b(\theta)\, db = dS \Rightarrow -J\, 2\pi\, b(\theta) \frac{db}{d\theta} = \frac{dS}{d\theta}$$

↑ less scattering when b becomes larger

$$\frac{db}{d\theta} < 0$$

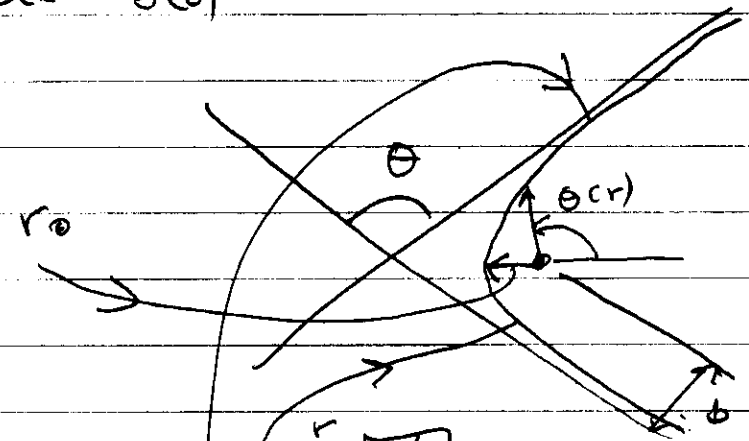


$$\Rightarrow \sigma(\theta)\, 2\pi J \sin\theta = -2\pi J\, b(\theta) \frac{db}{d\theta}$$

$$\Rightarrow \sigma(\theta) = -\frac{b(\theta)}{\sin\theta} \frac{db}{d\theta}$$

IV c) Rutherford cross-section

We consider a central potential. Need to calculate $b(\theta)$



$$L_z = b m v = b\sqrt{2mE}$$

$$\theta(r) = \pi + \int_{r_0}^{r} \sqrt{\frac{L_z^2}{2m}} \frac{dr}{r^2 \sqrt{E - V(r) - \frac{L_z^2}{2mr^2}}}$$

Scattering angle $-\theta = \pi - 2\,\theta(r=\infty)$

$$= -\pi + 2\int_{r_0}^{\infty} \sqrt{\frac{L_z^2}{2m}} \frac{dr}{r^2 \sqrt{E - V(r) - \frac{L_z^2}{2mr^2}}}$$