$$\Delta\phi = \frac{1}{\omega}\int_{-a}^{a}\frac{du}{\sqrt{u^2-a^2}}$$

$u_{min} = -a$

$u_{max} = a$

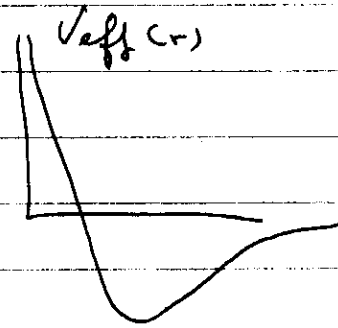
quadratic

$\Rightarrow u_{min} = -u_{max}$

$$u = a\cos\varphi$$

$$= \frac{2}{\omega}\,\frac{\pi}{2} = \frac{\pi}{\omega}$$

Next we consider $K<0$ and to get bound states, we need $\alpha < 0$.



Take $E \to 0 \Rightarrow u_{min} \simeq 0$

$u_{max} \sim \frac{1}{r_{max}}$

$$\frac{L_z^2}{2m}u_{max}^2 + \alpha u_{max}^{-K} = 0$$

$$\Rightarrow u_{max}^{2+K} = -\frac{\alpha m^2}{L_z^2}$$

$$\Rightarrow \Delta\varphi = \sqrt{\frac{L_z^2}{2m}}\int_{u_{min}\simeq 0}^{u_{max}}\frac{du}{\sqrt{-\alpha u^{-K} - \frac{L_z^2}{2m}u^2}}$$

$u' = \frac{u}{u_{max}}$

$$= \sqrt{\frac{L_z^2}{2m}}\,u_{max}\int_0^1\frac{du'}{\sqrt{-\alpha u_{max}^{-K}u'^{-K} - \frac{L_z^2}{2m}u_{max}^2 u'^2}}$$

$$= \int_0^1\frac{du'}{\sqrt{\underbrace{-\frac{\alpha 2m}{L_z^2}u_{max}^{-K-2}}_{=1}u'^{-K} - u'^2}}$$

$$= \int_0^1\frac{du'}{\sqrt{y^{-K}-y^2}} = \frac{\pi}{2+K}$$

other condition $\Delta\varphi = \frac{\pi}{\sqrt{2+K}}$

$\Rightarrow K = -1$