Then $V_{eff} = V_{eff}(u_0) + \frac{1}{2}(u-u_0)^2 V''_{eff}(u_0)$

$$\omega = \sqrt{\frac{V''_{eff}(r_0)}{L_z^2/m}} \Rightarrow \Delta\varphi = \frac{2\pi\,\omega^{-1}}{2} = \frac{2\pi}{2}\left[\sqrt{\frac{V''_{eff}(r_0)}{(L_z^2/m)}}\right]^{-1}$$

$$V''_{eff}(u_0) = \frac{L_z^2}{m} + U''(u_0)$$

$$V'_{eff}(u_0) = 0 = \frac{L_z^2}{m}u_0 + U'(u_0) = 0 \Rightarrow \frac{L_z^2}{m} = -\frac{U'(u_0)}{u_0}$$

$$\Rightarrow \frac{V''_{eff}(u_0)}{L_z^2/m} = \frac{\frac{L_z^2}{m} + U''(u_0)}{\frac{L_z^2}{m}} = \frac{-U' + u_0 U''}{-U'}$$

~~U = d~~ : <u>all</u> orbits can only be closed if $\Delta\phi$ or ω is independent of L_z^2/m. This is the case if ω does not depend on u_0.

$\Rightarrow u_0 U'' = U' \Rightarrow U$ is a homogeneous function of u_0

$$U = \alpha u^{-K} \Rightarrow \omega^2 = \frac{K + K(K+1)}{K} = K+2$$

$$\Rightarrow \Delta\varphi = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{K+2}}$$

h.o. gravity

closed if rational $\Rightarrow k = 2, -1, \frac{5}{4}, 7$

$$\Delta\phi = \int_{u_{min}}^{u_{max}} \frac{du\, L_z}{\left(2m\left(E - V_{eff}(u_0) + \frac{1}{2}(u-u_0)^2 V''_{eff}(u_0)\right)\right)^{\frac{1}{2}}}$$

$$= \frac{1}{\omega}\int_{u_{min}}^{u_{max}} \frac{du}{\sqrt{(u-u_{min})(u_{max}-u)}} \qquad \omega^2 \frac{L_z^2}{m}$$