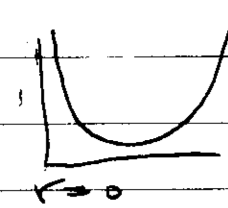
$\Delta\phi$ is a continuous function of E; so it is only possible that all orbits are closed if $\Delta\varphi$ does not depend on E

This means that we can study $E\to\infty$ and $E\to 0$ only

use $u=\frac{1}{r}$ as integration variable

$$\Rightarrow \Delta\phi = \int_{u_{min}}^{u_{max}} \frac{L_z\,du}{(2m(E-V_{eff}(u)))^{1/2}}$$

Let us consider $U(r)=\alpha r^K$ and take $K>0$



$E\to\infty \Rightarrow u_{min}\simeq 0$

$E = \frac{L_z^2}{2m}u_{max}^2$

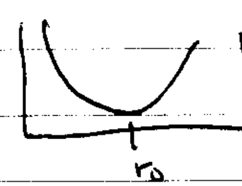
$$\Rightarrow \Delta\phi = \int_0^{u_{max}} \frac{L_z\,du}{\sqrt{2m}\left(\frac{L_z^2}{2m}u_{max}^2 - \frac{L_z^2}{2m}u^2 - \alpha r^K\right)^{1/2}}$$

$r=\frac{1}{u}$, $\alpha r^K = \alpha\frac{1}{u^K}$

$u_{max}\to\infty$

$$\Rightarrow \Delta\phi \simeq \int_0^{u_{max}} \frac{du}{\sqrt{u_{max}^2-u^2}} = \frac{\pi}{2}$$

Next we consider an almost circular orbit



$$\Delta\phi = \int_{u_{min}}^{u_{max}} \frac{du}{\left(\frac{2m}{L_z^2}(E-V_{eff}(u))\right)^{1/2}}$$

motion of particle in $V_{eff}(u)$ with mass $\frac{L_z^2}{m}$