

Example motion on a circle

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$$

constraint $F = x^2 + y^2 - R^2$

$$L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + \lambda F$$

$$EL \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \Rightarrow m \ddot{x} = 2\lambda x$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = 0 \Rightarrow m \ddot{y} = 2\lambda y$$

$$x^2 + y^2 = R^2 \Rightarrow 2x\dot{x} + 2y\dot{y} = 0$$

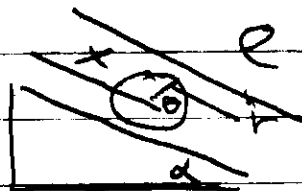
$$\Rightarrow 2\ddot{x}x + 2\dot{x}^2 + 2\ddot{y}y + 2\dot{y}^2 = 0$$

$$\Rightarrow m(\dot{x}^2 + \dot{y}^2) = -2\lambda(x^2 + y^2) = -2\lambda R^2$$

$$\Rightarrow \lambda = \frac{-m\dot{v}^2}{2R^2}$$

$$\text{Força} = 2\lambda \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{m\dot{v}^2}{R^2} \begin{pmatrix} x \\ y \end{pmatrix}$$

↑ centrifugal force

Example

constraint $r d\theta = dx$

$$T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} M r^2 \dot{\theta}^2$$

$$V = Mg(e-x)\sin\alpha$$

$$L = T - V = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} M r^2 \dot{\theta}^2 - Mg(e-x)\sin\alpha$$