$$\frac{d}{dt}\frac{\partial x_i}{\partial q_k} = \frac{\partial^2 x_i}{\partial q_k \partial q_\ell}\dot{q}_\ell + \frac{\partial}{\partial t}\frac{\partial x_i}{\partial q_k}$$

$$= \frac{\partial}{\partial q_k}\left(\frac{\partial \dot{x}_i}{\partial q_\ell}\dot{q}_\ell + \frac{\partial \dot{x}_i}{\partial t}\right) = \frac{\partial}{\partial q_k}\dot{\vec{x}}_i$$

$$\frac{d}{dt}\vec{x}_i$$

So we obtain $\ddot{\vec{x}}_i \frac{\partial \vec{x}_i}{\partial q_k} = \frac{d}{dt}\dot{\vec{x}}_i \frac{\partial \dot{\vec{x}}_i}{\partial \dot{q}_k} - \dot{\vec{x}}_i \frac{\partial}{\partial q_k}\dot{x}_i$

$$= \frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k}$$

$$T = \frac{1}{2}\sum m_i \dot{\vec{x}}_i^2$$

$\Rightarrow$ NE $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k}$ with

$$L = T - V + \sum_\ell x_\ell F_\ell$$