II h) Constraints

holonomic constraints $f_e(x_1, \dots, x_N, t) = 0$

$e = 1, \dots, k$

example: motion on a circle $x^2 + y^2 = R^2$

nonholonomic constraints $f_e(x_1 \dots x_N, \dot{x}_1, \dots, \dot{x}_N, t) = 0$

if the differential equation can be integrated the constraints are holonomic

II i) Newton eqs and constraints

$$f(\vec{x}, t) = 0 \quad \Rightarrow \quad NE = m\ddot{\vec{x}} = \vec{F} + \underbrace{\vec{C}}_{C}$$

constraint force to keep particle on surface

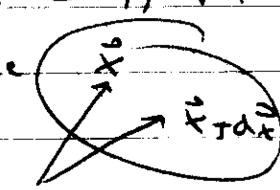
Let us take $\vec{x} = (x_1, x_2, x_3)$

the we have 6 unknowns ($\vec{x}$ and $\vec{C}$)

and only 4 eqs.

However, the constraint force is always perpendicular to the surface $\Rightarrow \vec{C} \parallel \vec{\nabla} f$

proof Let $\vec{x}$ and $\vec{x} + d\vec{x}$ on surface



then

$$f(\vec{x} + d\vec{x}) = f(\vec{x}) = 0$$

$$" f(\vec{x}) + d\vec{x}\, \vec{\nabla} f(\vec{x}) \Rightarrow \vec{\nabla}(f(\vec{x})) \perp \text{surface}$$

$\Rightarrow$ only force $\perp$ surface is needed to keep a particle on a surface

$$\Rightarrow \vec{C} = \lambda \vec{\nabla} f$$