Noether's theorem

$$\lim_{\alpha \to 0} \frac{\partial L_\alpha}{\partial \dot{q}_\mu(\alpha)} \frac{\partial q_\mu(\alpha)}{\partial \alpha} = \text{conserved}$$

Example $L = \sum m x_\mu^2$

rotation about z-axis $x_\mu(\theta) = x_\mu \cos\theta + y_\mu \sin\theta$

$y_\mu(\theta) = -x_\mu \sin\theta + y_\mu \cos\theta$

$L = \sum m_\mu (\dot{x}_\mu(\theta))^2$ does not depend on θ

$$\left.\frac{\partial x_i}{\partial \theta}\right|_{\theta=0} = y_i \qquad \left.\frac{\partial y_i}{\partial \theta}\right|_{\theta=0} = -x_i$$

$$\Rightarrow \sum_\mu m_\mu (\dot{x}_\mu y_\mu - \dot{y}_\mu x_\mu) = \text{constant}$$

$\frac{\partial L}{\partial \dot{x}_\mu}$ $\frac{\partial L}{\partial \dot{y}_\mu}$

Conservation of angular momentum

$x_\mu = \cos\theta\, x_\mu(0) - \sin\theta\, y_\mu(0)$ $\quad y_\mu = \sin\theta\, x_\mu(0) + \cos\theta\, y_\mu(0)$

$\dot{x}_\mu = \cos\theta\, \dot{x}_\mu(0) - \sin\theta\, \dot{\theta}\, x_\mu(0) - \sin\theta\, \dot{y}_\mu(0) - \cos\theta\, \dot{\theta}\, y_\mu(0)$

$\dot{y}_\mu = \sin\theta\, \dot{x}_\mu(0) + \cos\theta\, \dot{\theta}\, x_\mu(0) + \cos\theta\, \dot{y}_\mu(0) - \sin\theta\, \dot{\theta}\, y_\mu(0)$

$\dot{x}_\mu^2 + \dot{y}_\mu^2 = -\dot{\theta}\, \dot{x}_\mu(0)\, y_\mu(0) + \dot{\theta}\, x_\mu(0)\, \dot{y}_\mu(0) + \dot{x}_\mu^2(0) + \dot{y}^2(0)$

$+ \dot{\theta}^2 x_\mu^2(0) + \dot{\theta}^2 y_\mu^2(0) \quad \Rightarrow \frac{\partial L}{\partial \theta} = 0$