Example: central force: \( L = \frac{1}{2m} (\dot{r}^2 + r^2 \dot{\theta}^2) - V(r) \)

\( \theta \) in cyclic \( \Rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} = 0 \)

\( \uparrow \)

Angular momentum

IIq) Noether's Theorem

This theorem relates directly symmetries to conserved quantities.

We consider a coordinate transformation

\( q_k (x) = \mathcal{F}_k (q_1, \ldots, q_n, x) \)

such that \( L = L (q_k(x), \dot{q}_k(x)) \) does not depend on \( x \).

Then \( x \) is a cyclic coordinate

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = 0 \Rightarrow \frac{\partial L}{\partial \dot{x}} = \text{constant} \]

\[ \frac{\partial L}{\partial \dot{q}_k} \frac{\partial \mathcal{F}_k}{\partial x} + \frac{\partial L}{\partial q_k} \frac{\partial \mathcal{F}_k}{\partial \dot{x}} = \text{constant} \]

\[ \Rightarrow 0 \]

\[ \Rightarrow \frac{\partial L}{\partial q_k} \frac{1}{\mathcal{F}_k} = \text{constant} \]