Angular momentum $\vec{M} = \sum_k \vec{r}_k \times \vec{p}_k$

conserved for a closed system

If $r_{kj} = r_k' + a$ Then

$$\vec{M} = \vec{M}' + \vec{a} \times \vec{P}$$

Angular momentum in cylinder coordinates

$$M_z = \sum_k m_k (x_k \dot{y}_k - y_k \dot{x}_k)$$

$$= \sum_k m_k \left(r_k \cos\varphi \left(\dot{r}_k \sin\varphi_k + r_k \dot{\varphi} \cos\varphi_k \right) - r_k \sin\varphi \left(\dot{r}_k \cos\varphi_k - r_k \dot{\varphi} \sin\varphi_k \right) \right)$$

$$= \sum_k m_k r_k^2 \dot{\varphi}_k$$

II f) Cyclic coordinates

Suppose that the Lagrangian does not depend on q_p, then q_p is called a cyclic coordinate

$$\Rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_p} = \frac{\partial L}{\partial q_p} = 0$$

$$\Rightarrow \frac{\partial L}{\partial \dot{q}_p} \text{ is a constant of motion}$$