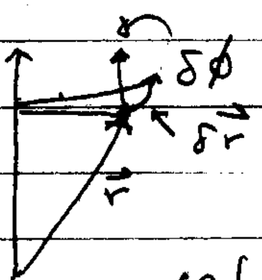
$$\Rightarrow E = \frac{1}{2}\mu v_0^2 + \underbrace{\frac{1}{2}\sum_k m_k v_k'^2 + U}_{U_{int}}$$

II e) Angular momentum

We consider an infinitesimal rotation



$$\vec{\delta r} = \vec{\delta\phi} \times \vec{r}$$

velocity is also a vector $\Rightarrow$ $\delta\vec{v} = \delta\phi \times \vec{v}$

isotropy of space: Lagrangian is invariant under rotations

$$\delta L = \frac{\partial L}{\partial r_k}\delta r_k + \frac{\partial L}{\partial \dot{r}_k}\delta \dot{r}_k$$

$$= \underset{\dot{\vec{p}}_k}{\underset{\parallel}{\frac{\partial L}{\partial r_k}}}\,\delta\phi \times \vec{r}_k + \underset{\vec{p}_k}{\underset{\parallel}{\frac{\partial L}{\partial \dot{r}_k}}}\,\underset{\delta\phi \times \dot{r}_k}{\underset{\parallel}{\delta \dot{\vec{r}}_k}} = 0$$

$$\Rightarrow \delta L = \dot{\vec{p}}_k \cdot \vec{\delta\phi} \times \vec{r}_k + \vec{p}_k \cdot \vec{\delta\phi} \times \dot{r}_k$$

$$= \delta\phi \cdot (\vec{r}_k \times \dot{\vec{p}}_k + \dot{\vec{r}}_k \times \vec{p}_k)$$

$$= \vec{\delta\phi} \cdot \frac{d}{dt}(\vec{r}_k \times \vec{p}_k) = 0$$

arbitrary $\Rightarrow$ $\frac{d}{dt}(\vec{r}_k \times \vec{p}_k) = 0$