II d) center of mass

transformation of momentum under Galilei transformations

$$\vec{r}' = \vec{r} + \vec{v}_0 t$$

$$\Rightarrow \vec{P} = \sum_\alpha m_\alpha \vec{v}_\alpha = \sum_\alpha m_\alpha (\vec{v}_\alpha' - \vec{v}_0)$$

$$= \vec{P}' - \vec{v}_0 \sum_\alpha m_\alpha$$

zero momentum frame $\quad \vec{P}' = 0$

$$\Rightarrow \vec{v}_0 = -\frac{\vec{P}}{\sum_\alpha m_\alpha} = -\frac{\sum m_\alpha \vec{v}_\alpha}{\sum m_\alpha}$$

$$= -\frac{d}{dt} \frac{\sum m_\alpha \vec{r}_\alpha}{\sum m_\alpha}$$

$\equiv R = \cancel{\text{center of mass}}$

The center of mass moves uniformely for a closed system

Internal energy: energy in the $p=0$ frame

Total energy: $E_{tot} = E_{int} + \frac{1}{2}\mu V_0^2$

$$\mu = \sum_\alpha m_\alpha$$

proof $\quad E = \frac{1}{2} \sum_\alpha m_\alpha v_\alpha^2 + U$

$$= \frac{1}{2} \sum_\alpha m_\alpha (\vec{v}_\alpha' - \vec{v}_0)^2 + U$$

$$= \frac{1}{2} \sum_\alpha m_\alpha v_\alpha'^2 - v_0 \underbrace{\sum_\alpha m_\alpha v_\alpha'}_{=0 \text{ in } p'=0 \text{ frame}} + U + \frac{1}{2}\mu v_0^2$$