The KAM theorem

All rational tori are unstable. This is not the case for irrational tori.

An instability occurs if \((\mathbf{v}_0, \mathbf{m}) = 0\) for some set of integers \(m, n\) rotation frequencies of the unperturbed system.

So we have to stay away from frequencies for which \((\mathbf{v}_0, \mathbf{m}) = 0\), so that

\[
\frac{L}{\frac{d}{d, \mathbf{m}, \mathbf{v}_0}} \mathbf{K} d(\mathbf{v}_0) = \text{i} m \mathbf{v}_0^0
\]

converges for \(L \to 0\).

We wish to exclude all frequencies inside a band \(\mathbf{m}, \mathbf{v} \mid, \mathbf{v} \leq \delta\), but we also wish to end up with a finite \(\langle 0, \mathbf{m}_1 \rangle = 0\) piece of phase space.

Consider first the case that \(H_0\) is a linear function of the \(J_k\), i.e., \(H_0 = J_1 + J_2 + J_3\)

\[
\begin{align*}
J_1 &= 1 \\
J_2 &= 2
\end{align*}
\]

Small denominator relation \(m_1 + 2m_2 = 0\).

We cannot adjust the \(J_k\) to stay away from the small denominator.