What happens for $\varepsilon > 0$?

- $Z_{\varepsilon}$ is an area preserving map.

$$
(\phi, \psi) \rightarrow \left( \frac{\varepsilon}{2} \phi + \frac{\varepsilon}{2} \sin \phi + \frac{\varepsilon}{2}, \frac{\varepsilon}{2} \sin \phi + \frac{\varepsilon}{2} \right)
$$

$$
(\phi + d\phi, \psi) \rightarrow \left( \frac{\varepsilon}{2} \phi + \frac{\varepsilon}{2} \sin \phi + \frac{\varepsilon}{2} \cos \phi \; d\phi + \frac{\varepsilon}{2}, \frac{\varepsilon}{2} \sin \phi + \frac{\varepsilon}{2} \cos \phi \; d\phi + \frac{\varepsilon}{2} \right)
$$

As $d\phi \rightarrow 0$:

$$
\int_{-\frac{\varepsilon}{2}}^{\frac{\varepsilon}{2}} \frac{\varepsilon}{2} \sin \phi \; d\phi = -d\phi
$$

- $Z_{\varepsilon}^{-1}(\phi, \psi)$ is continuous as a function of $\varepsilon$.
- It maps a circle to a deformed closed curve with the same area.

For larger $\varepsilon$, the angles move faster and for smaller $\varepsilon$, they move slower. We can find a curve such that the $\phi$ values are not changed by the map.