II c) Momentum

homogeneity of space: properties of a closed system are invariant under translations

$$\vec{r} \to \vec{r} + \vec{\varepsilon} \Rightarrow \delta L = \sum_a \frac{\partial L}{\partial r_a} \delta r_a + \varepsilon \sum_a \frac{\partial L}{\partial r_a}$$

↑ sum over particles

$$\delta L = 0 \Rightarrow \sum_a \frac{\partial L}{\partial r_a} = 0$$

$$EL \Rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{r}_a} = 0$$

total momentum of a system $\vec{P} = \sum_a \frac{\partial L}{\partial \dot{r}_a}$

$$= \sum_a m \vec{v}_a$$

$$\frac{\partial L}{\partial r_a} = -\frac{\partial U}{\partial r_a} = \text{force on particle } a$$

$$\Rightarrow \sum_a F_a = 0 \quad \text{(Newton's third law)}$$

generalized momenta $\frac{\partial L}{\partial \dot{q}_K} = p_K$

generalized Forces $F_K = \frac{\partial L}{\partial q_K}$