$$\Rightarrow \frac{dL}{dt} = \frac{\partial L}{\partial q_k}\dot{q}_k + \frac{\partial L}{\partial \dot{q}_k}\ddot{q}_k$$

$$= \dot{q}_k \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} + \frac{\partial L}{\partial \dot{q}_k}\ddot{q}_k$$

$$= \frac{d}{dt}\left(\dot{q}_k \frac{\partial L}{\partial \dot{q}_k}\right)$$

$$\Rightarrow \frac{d}{dt}\left[L - \dot{q}_k \frac{\partial L}{\partial \dot{q}_k}\right] = 0$$

<u>energy</u> $E \equiv \sum_k \dot{q}_k \frac{\partial L}{\partial \dot{q}_k} - L$

<u>conservative systems</u>: mechanical systems for which the energy is conserved

$L = T - U$ T is a quadratic function of velocities

$$\Rightarrow \sum_k \dot{q}_k \frac{\partial L}{\partial \dot{q}_k} = \sum_k \dot{q}_k \frac{\partial T}{\partial \dot{q}_k} = 2T$$

$$\Rightarrow E = 2T - T + U = T + U$$

of cartesian $E = \sum_k \frac{1}{2} m \dot{q}_k^2 + U(q_1 \dots q_s)$