II a) Integrals of motion

$F(q_k, \dot{q}_k)$ that stays constant during time evolution.

special integrals of motion: conserved quantities
they are additive

If there are s degrees of freedom there are at most $2s-1$ constants of motion

<u>proof</u> equations of motion do not depend on the initial time

initial conditions $\dot{q}_1(t_0) = F_1(q_k, \dot{q}_k, t_0)$, $q_1(t_0) = C_2(q_k, \dot{q}_k, t_0)$
etc.

$2s$ initial conditions
eliminate $t_0 \Rightarrow 2s+1$ integrals of motion

II b) Energy

homogeneity of time $\Rightarrow$

~~total time derivative~~ of Lagrangian
~~is constant~~ Lagrangian does not
depend explicitly $\frac{dL}{dt} = 0$ on time

$$\frac{\partial L}{\partial t} = 0 \quad \text{or} \quad L = L(q_k, \dot{q}_k)$$