Canonical transformation

\[
q_i = \frac{\partial F}{\partial p_i}, \quad p_i = -\frac{\partial F}{\partial q_i}, \quad H = H + \frac{\partial F}{\partial t}
\]

PB are invariant under canonical transformations

Lagrange bracket:

\[\{a, \varphi\} = \sum_i \frac{\partial a}{\partial q_i} \frac{\partial \varphi}{\partial p_i} - \frac{\partial a}{\partial p_i} \frac{\partial \varphi}{\partial q_i}\]

HJ eq.

\[\frac{\partial S}{\partial t} + H(q, p, \frac{\partial S}{\partial q}, t) = 0\]

\[S = S_0 - Et\] separation of variables

\[L = \frac{1}{2} m \dot{x}^2 + e A \cdot \dot{x} - e \Phi\]

gauge invariance

Liouville theorem

\[d\Omega = dq_1 \wedge dq_2 \wedge dp_1 \wedge dp_2\]

\[d\Omega\] is invariant under canonical transformations

\[\sum \delta q_i \delta p_i\] is invariant under canonical transformations

Poincaré section - area preserving

\[\dot{q}_2 = -\frac{\partial H}{\partial p_2}, \quad \dot{p}_2 = \frac{\partial H}{\partial q_2}\]

Lyapunov exponents

cat map

Kicked rotor

Ergodic mixing, \(\alpha\)-system, Bernoulli

Poincaré recurrence