Homework Set # 9, due November 12, 2007

1. a) Show that if $u$ is a scalar function, i.e. it only depends $\vec{r}^2$, $\vec{p}^2$ and $\vec{r}.\vec{p}$, then

$$[u, M_k] = 0, \quad M_k \text{ is angular momentum.}$$

b) Show that if $F$ is a vector function, i.e.

$$F_k = ur_k + vp_k,$$

with $u$ and $v$ scalar functions, then

$$[F_i, M_j] = -\epsilon_{ijk}F_k$$

2. The transformation between two sets of coordinates is given by

$$Q = \log(1 + q^{1/2} \cos p),$$
$$P = 2(1 + q^{1/2} \cos p)q^{1/2} \sin p.$$  

(a) Show that $Q$ and $P$ are canonical variables if $q$ and $p$ are.

(b) Show that the generating function of this canonical transformation is given by

$$F = -(e^Q - 1)^2 \tan p.$$  

3. A uniform bar of mass $M$ and length $2l$ is suspended from one end by a spring with spring constant $k$. The bar can swing freely in one vertical plane and the spring can only move in the vertical direction. Find the Hamiltonian of this system and obtain the Hamilton equations of motion.

4. Consider a particle in 3 dimensions, $x$, $y$, $z$ with momenta $p_x$, $p_y$, $p_z$ and Hamiltonian

$$H = \frac{1}{2m}[(p_x + ay)^2 + p_y^2 + (p_z + bz)^2],$$

where $m$, $a$ and $b$ are constants.

(a) Find the equations of motion.

(b) Find the Lagrangian.

(c) Solve the equations of motion for $x, y$ and $z$ as function of $t$ and given initial conditions.