4) $H = \frac{1}{2m}(p_x + ay)^2 + \frac{p_y^2}{2m} + \frac{1}{2m}(p_z + bz)^2$

a) $\dot{p}_x = -\frac{\partial H}{\partial x} = 0$ $\qquad \dot{x} = \frac{\partial H}{\partial p_x} = \frac{1}{m}(p_x + ay)$

$\dot{p}_y = -\frac{\partial H}{\partial y} = -\frac{a}{m}(p_x + ay)$ $\qquad \dot{y} = \frac{p_y}{m}$

$\dot{p}_z = -\frac{\partial H}{\partial z} = -\frac{b}{m}(p_z + bz)$ $\qquad \dot{z} = \frac{p_z + bz}{m}$

b) $L = -H + p_\alpha \dot{q}_\alpha$

$= -\frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\dot{y}^2 - \frac{1}{2}m\dot{z}^2 + \underset{(m\dot{x} - ay)}{\underset{\shortparallel}{p_x}}\dot{x} + \underset{m\dot{y}}{\underset{\shortparallel}{p_y}}\dot{y} + \underset{(m\dot{z} - bz)}{\underset{\shortparallel}{p_z}}\dot{z}$

$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m\dot{z}^2 - ay\dot{x} - bz\dot{z}$

c) EL eqs

$x: \quad m\ddot{x} = a\dot{y}$

$y: \quad m\ddot{y} = -a\dot{x}$

$z: \quad m\ddot{z} = b\dot{z} - b\dot{z} = 0 \quad \Rightarrow z(t) = z_0 + v_{z_0} t$

$m\ddot{y} = -a\dot{x} \quad \Rightarrow \quad m\dot{y} = -a(x - x_0)$ ← integration constants

$m\ddot{x} = a\dot{y} \qquad\qquad m\dot{x} = -a(y - y_0)$

$\Rightarrow m(\dot{x} + \dot{y}) = -a(x + y - x_0 - y_0)$

$\Rightarrow (x+y)(t) = (x_0 + y_0)(e^{-\frac{a}{m}t} + 1)$

$m(\dot{y} - \dot{x}) = -a(x - x_0 - y + y_0)$

$\Rightarrow (x - y)(t) = (x_0 - y_0)(e^{+\frac{a}{m}t} - 1)$

From this we find $x(t)$ and $y(t)$