

1a) $u = u(r^i, p^i, r, p)$
 then $[u, M] = 0$

M is angular momentum

$$[u, \pi_k] = -\partial_{p_i} u \partial_{x_i} \pi_k + \partial_{p_i} M_k \partial_{x_i} u$$

$$\pi_k = \epsilon_{k\ell m} x_\ell p_m$$

$$\Rightarrow [u, M_k] = -\partial_{p_i} u \epsilon_{kim} p_m + \partial_{x_i} u \epsilon_{k\ell i} x_\ell$$

$$= -\epsilon_{kim} p_m \partial_{p_i} u + \epsilon_{k\ell i} x_\ell \partial_{x_i} u$$

$$\epsilon_{kim} p_m p_i = 0 \quad \Rightarrow \text{only the 3 arguments can contribute}$$

$$= (\epsilon_{kim} p_m x_i - \epsilon_{k\ell i} x_\ell p_i) \partial_i u = 0$$

1b) $F_i = u r_i + \sigma p_i$

then $[F_i, \pi_j] = +\epsilon_{ijk} F_k$

- show it separately for each term

$$[u r_i, \pi_j] = [u, \pi_j] r_i + u [r_i, \pi_j]$$

$$[r_i, \pi_j] = +\partial_{r_k} r_i \partial_{p_k} \pi_j = +\delta_{ik} \epsilon_{j\ell k} r_\ell = +\epsilon_{j\ell i} r_\ell$$

$$[p_i, \pi_j] = -\partial_{p_k} p_i \partial_{r_k} \pi_j = -\delta_{ik} \epsilon_{j\ell m} p_m \delta_{\ell k} = -\epsilon_{j\ell i} p_\ell$$

$$\Rightarrow [F_i, \pi_j] = +\epsilon_{ijk} F_k$$