# Homework set VII

1) Show that an upward pendulum is stable for $a^2\gamma^2 > 2gl$ if its support point oscillates as $x = a\cos\gamma t$ and $\gamma \gg \sqrt{g/l}$ (see problem 1, §30 of LL)

2) A uniform disk of mass $M$ and radius $a$ rolls without slipping in a fixed circular trough of radius $b > a$. The gravitatial accelleration is $g$. Find the Lagrangian of the system and determine the frequency of small oscillations of the disk about its equilibrium position.

3) Consider the diff. eq. $\ddot{x} + \omega^2 x = -\frac{3}{4}a^2\cos\omega t - \frac{1}{4}a^3\cos 3\omega t$ with initial conditions $x_1(0) = 0$ $\dot{x}_1 = 0$

a) Find the general solution of the homogeneous equation

b) Find a special solution of the inhomogeneous equation of the form $x = At\sin\omega t + B\cos\omega t$

c) The general solution is given by the sum of a) and b). Determine the integration constants from the initial cond.

4) Consider the anharmonic oscillator with $V(x) = x^2 + \varepsilon x^4$ $\varepsilon \ll 1$ and $m = 1$

a) Obtain an expression for the period of the oscillator.

b) Calculate the period to $\mathcal{O}(\varepsilon)$. Show that your result agrees with the formula we found in class ($\omega_0\omega_1 = \frac{3}{8}a^2$, see LL 28.13)