Homework set VII

1) Show that an upward pendulum is stable for $\frac{4}{9}g > 2g\ell$
if its support point oscillates as $x = x \cos \omega t$ and
$\omega^2 > \frac{g}{b}$ (see problem 1, §3.2 of LL).

2) A uniform disk of mass $M$ and radius $a$ rolls without
slipping in a fixed circular trough of radius $b > a$
The gravitational acceleration is $g$. Find the Lagrangian
of the system and determine the frequency of small
oscillations of the disk about its equilibrium position.

3) Consider the diff. eq. $\ddot{x} + \omega^2 x = -23 \frac{a^2}{b^3} \cos \omega t$
with initial conditions $x(0) = 0$, $\dot{x}(0) = 0$

a) Find the general solution of the homogeneous equation
b) Find a special solution of the inhomogeneous equation
of the form $x = f \sin \omega t + B \cos \omega t$
c) The general solution is given by the sum of a) and b).
Determine the integration constants from the initial cond.

4) Consider the anharmonic oscillator with
$V(x) = x^2 + \varepsilon x^4$, $\varepsilon \ll 1$ and $m = 1$

a) Obtain an expression for the period of the oscillator.
b) Calculate the period to $O(\varepsilon)$. Show that
your result agrees with the formula we found
in class ($\omega_{an} = \frac{\pi}{8} a^2$, see LL 28.13).