

Solutions of homework set #6

11 ~~$\frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} k x_1^2 - \frac{1}{2} k x_2^2 - \frac{1}{2} 3k (x_1 - x_2)^2$~~

$$L = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} k x_1^2 - \frac{1}{2} k x_2^2 - \frac{1}{2} 3k (x_1 - x_2)^2$$

$$m = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \quad K_{ij} = k \begin{pmatrix} 4 & -3 \\ -3 & 4 \end{pmatrix}$$

Eigenvalue equation

$$\det \left[\omega^2 m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - k \begin{pmatrix} 4 & -3 \\ -3 & 4 \end{pmatrix} \right] = 0$$

$$\Rightarrow m \omega^2 = 4k \pm 3k$$

$$2) \quad \sigma_{1x} = l \dot{\theta}_1 \cos \theta_1, \quad \sigma_{1y} = l \dot{\theta}_1 \sin \theta_1$$

$$\sigma_{2x} = \sigma_{1x} + l \dot{\theta}_2 \cos \theta_2, \quad \sigma_{2y} = \sigma_{1y} + l \dot{\theta}_2 \sin \theta_2$$

$$\Rightarrow L = T - V = \frac{1}{2} m l^2 \dot{\theta}_1^2 + \frac{1}{2} m l^2 (\dot{\theta}_1 \cos \theta_1 + \dot{\theta}_2 \cos \theta_2)^2$$

$$+ \frac{1}{2} m l^2 (\dot{\theta}_1 \sin \theta_1 + \dot{\theta}_2 \sin \theta_2)^2$$

$$+ m g l \cos \theta_1 + m g l (\cos \theta_1 + \cos \theta_2)$$

Small oscillations $\Rightarrow \cos \theta = 1 - \frac{1}{2} \theta^2, \sin \theta = \theta$

$$\Rightarrow L = \frac{1}{2} m l^2 \dot{\theta}_1^2 + \frac{1}{2} m l^2 \dot{\theta}_2^2 + \frac{1}{2} m l^2 \dot{\theta}_1 \dot{\theta}_2 + m g l \left(3 - \frac{2}{2} \theta_1^2 - \frac{\theta_2^2}{2} \right)$$

Eqs. of motion:

$$m l^2 \ddot{\theta}_1 + \frac{1}{2} m l^2 \ddot{\theta}_2 + 2 m g l \theta_1 = 0$$

$$m l^2 \ddot{\theta}_2 + \frac{1}{2} m l^2 \ddot{\theta}_1 + m g l \theta_2 = 0$$