Homework set #5

1) Obtain the Rutherford formula for a repulsive Coulomb potential.

2) Consider scattering for the inverted oscillator potential $V = -\frac{1}{2} K(x_1^2 + x_2^2)$

a) Show that all orbits are asymptotic to straight lines passing through the origin.

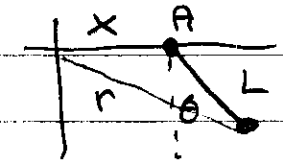
b) Therefore it is not possible to define an impact parameter, but a scattering angle can be defined. If B is the distance of closest approach, find the scattering angle as a function of B and E.

3) Consider the h.o. $H = \frac{1}{2}\omega^2(x_1^2 + x_2^2) + \frac{1}{2}(p_1^2 + p_2^2)$

a) Show that in addition to $H = E$ and $|\vec{M}| = x_1 p_2 - p_2 x_1$ there is a 3rd conserved quantity $A_{ij} = p_i p_j + \omega^2 x_i x_j$

b) Use A_{ij} to solve the eqs. of motion without integration (Hint: consider $x_i A_{ij} x_j$). Interpret the solution.

4) A pendulum bob of mass m is suspended by a weightless rod of length L. The support point A moves as $x = a\cos\omega t$



a) Find the Lagrangian and write out the Lagrange eqs.

b) Show that for small θ the motion reduces to that of a forced harmonic oscillator. Find the steady state motions. Discuss the behavior of the amplitude as a function of the forcing frequency.