

$$V = -\frac{R}{2}(x_1^2 + x_2^2)$$

According to problem 3  
the classical trajectories are given by

$$x_i (H \delta_{ij} - \frac{1}{2} A_{ij}) x_j = \frac{|P|^2}{2} \quad (1)$$

$$A_{11} = 2E_1 = -k x_1^2 + P_1^2$$

$$A_{22} = 2E_2 = -k x_2^2 + P_2^2$$

$$A_{12} = -k x_1 x_2 + P_1 P_2$$

We have to show that (1) represent a  
hyperbola with asymptotes through  $x_1 = x_2 = 0$ .

We only have to show that

$E \delta_{ij} - \frac{1}{2} A_{ij}$  has one positive and one  
negative eigenvalue

$$\det(E - \frac{1}{2} A_{ij} - \lambda) \Rightarrow \dots$$

$$x^2 - \lambda(E_1 + E_2) + E_1 E_2 - \frac{1}{4}(-k x_1 x_2 + P_1 P_2)^2 = 0$$

$\Rightarrow$

$$\Rightarrow -\frac{k}{4}(x_1 P_2 - x_2 P_1)^2 + x^2 - \lambda(E_1 + E_2) = 0$$

$$E - \lambda = \pm \frac{E_1 + E_2 \pm \sqrt{(E_1 + E_2)^2 + k(x_1 P_2 - x_2 P_1)^2}}{2}$$

note

$\Rightarrow$  We have one positive and one negative eigenvalue  
For  $k < 0$ ,  $x_1 > 0$  and  $x_2 > 0$ ; then ellipse