Homework set #4, due October 1 2007)

1) a) For $V(r) = -\frac{\alpha}{r}$ show that

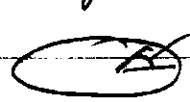
$\vec{A} = \vec{p} \times \underbrace{\vec{L}}_{\vec{r} \times \vec{p}} - m\alpha \frac{\vec{r}}{r}$ is conserved.

b) Use this to solve the Kepler problem algebraically. (hint: consider $\vec{r} \cdot \vec{A}$)

2) A point like mass m is moving under the influence of an attractive force $\vec{F} = -f(r)\hat{r}$, $f(r) > 0$

a) Show that $\vec{L}$ and L_z with respect to the center of attraction are constants of motion

b) In terms of u and φ show that the diff. eqn of motion is $u + \frac{d^2u}{d\varphi^2} = -\frac{m}{\ell^2 u^2} f\left(\frac{1}{u}\right)$



c) For $f(r) = kr^n$, $k > 0$ show that stable circular orbits only exist for $n > -3$.

3) Consider the potential $V(r) = k|\vec{r}|$ felt by mass m

a) Find the condition for a particle with angular momentum ℓ to move in a stable circular orbit

b) Now increase the energy slightly beyond a circular orbit while keeping the angular momentum fixed. How much does the angular position change in one period of radial oscillation.

4) Find the orbit $\varphi(r)$ for the central potential $U(r) = -\frac{\alpha}{r} + \frac{\beta}{r^2}$ for all ranges of energy ($\alpha, \beta > 0$).