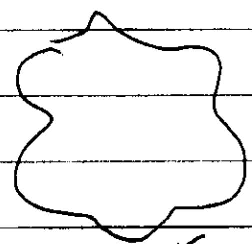
3) $V = k r$

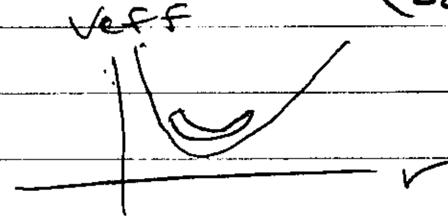
$$V_{eff} = \frac{L_z^2}{2mr^2} + kr$$

a) minimum $-\frac{L_z^2}{mr^3} + k = 0 \Rightarrow r^3 = \frac{km}{L_z^2}$

$$r = \left(\frac{km}{L_z^2}\right)^{1/3}$$

b)





$$\Delta\phi = 2\int_{r_{min}}^{r_{max}} \frac{L_z\,dr}{r^2\left(2m\left(E - V_{eff}\right)\right)^{\frac{1}{2}}}$$

(see lecture notes p.27)

$$= 2\int_{r_{min}}^{r_{max}} \frac{L_z\,dr}{r^2\left(2m\left(E - kr - \frac{L_z^2u^2}{2m}\right)\right)^{1/2}}$$

$$= 2\int_{u_{min}}^{u_{max}} \frac{L_z\,du}{\left(2m\left(E - k/u - \frac{L_z^2u^2}{2m}\right)\right)^{1/2}} = 2\int_{u_{min}}^{u_{max}} \frac{L_z\,du\,\sqrt{u}}{\sqrt{2m}\left(Eu - k - \frac{L_z^2u^3}{2m}\right)^{1/2}}$$

near minimum

$$\frac{L_z^2u^2}{2m} + \frac{k}{u} \simeq \frac{L_z^2u_0^2}{2m} + \frac{k}{u_0} + (u-u_0)\left(\frac{L_z^2u_0}{m} - \frac{k}{u_0^2}\right) + \frac{1}{2}(u-u_0)^2\left(\frac{L_z^2}{m} + \frac{2k}{u_0^3}\right)$$

$$k = \frac{L_z^2}{m}u_0^3$$

$$\Rightarrow \Delta\varphi = 2\cancel{\sqrt{\frac{L_z^2}{2m}}}\int_{u_{min}}^{u_{max}} \frac{du}{\cancel{\sqrt{\frac{L_z^2}{2m}}}\sqrt{3}\sqrt{(u_{max}-u)(u-u_{min})}}$$

$$= \frac{2\pi}{\sqrt{3}}$$