2a) $\vec{F} = -f(r)\,\hat{r} \Rightarrow V(r) = +\int^{r} r' f(r')\,dr'$

$$-\vec{\nabla} V = -r\, f(r)\, \frac{\vec{r}}{r} = -f(r)\,\hat{r}$$

$\Rightarrow$ $V(r)$ is spherically symmetric $\Rightarrow$ $\vec{L}$ is conserved (see lecture)

2b) Effective potential $V_{eff} = \frac{L_z^2}{2mr^2} + V(r)$

equation of motion $m\ddot{r} = -\vec{\nabla} V_{eff} = \frac{L_z^2}{mr^3} - f(r)$

$$L_z = mr^2\dot{\varphi} \Rightarrow \frac{dr}{d\varphi} = \frac{dr}{dt}\frac{dt}{d\varphi}$$

$$\Rightarrow \frac{dr}{dt} = \frac{L_z}{mr^2}\frac{dr}{d\varphi}$$

$$\frac{d^2r}{dt^2} = \frac{L_z}{m}\frac{-2}{r^3}\frac{L_z}{mr^2}\left(\frac{dr}{d\varphi}\right)^2 + \left(\frac{L_z}{mr^2}\right)^2\frac{d^2r}{d\varphi^2}$$

$$= \frac{-2L_z^2}{m r^5}\left(\frac{dr}{d\varphi}\right)^2 + \left(\frac{L_z}{mr^2}\right)^2\frac{d^2r}{d\varphi^2}$$

$$r = \frac{1}{u} \Rightarrow \frac{dr}{d\varphi} = -\frac{1}{u^2}\frac{du}{d\varphi}$$

$$\frac{d^2r}{d\varphi^2} = +\frac{2}{u^3}\left(\frac{du}{d\varphi}\right)^2 - \frac{1}{u^2}\frac{d^2u}{d\varphi^2}$$

$$\Rightarrow m\left[\cancel{-\frac{2L_z^2}{m}u\left(\frac{du}{d\varphi}\right)^2} + \cancel{\frac{L_z^2}{m}(+2)\,u\left(\frac{du}{d\varphi}\right)^2} - \left(\frac{L_z}{m}\right)^2 u^2\frac{d^2u}{d\varphi^2}\right]$$

$$= \frac{L_z^2}{m}u^3 - f\left(\frac{1}{u}\right)$$

$$\Rightarrow u + \frac{d^2u}{d\varphi^2} = -\frac{m}{L_z^2 u^2} f\left(\frac{1}{u}\right)$$