Solutions of homework set #4

1a) $\dot{\vec{p}} = -V'(r)\hat{r} \rightarrow \dot{\vec{p}} \times \vec{L} = -V'(r)\hat{r} \times (\vec{r} \times \dot{\vec{p}})$

$= -V'(r)\, m\, r\, \hat{r} \times (\vec{r} \times \dot{\hat{r}})$

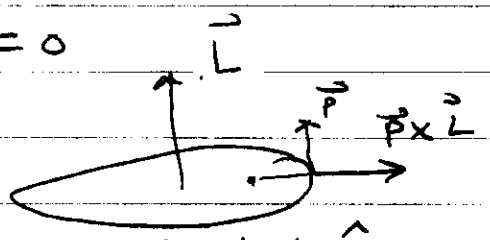
$\vec{a} \times (\vec{b} \times \vec{c}) = -(\vec{a}\cdot\vec{b})\vec{c} + (\vec{a}\cdot\vec{c})\vec{b}$

$\hat{r} \cdot \dot{\hat{r}} = 0$

$\Rightarrow \dot{\vec{p}} \times \vec{L} = V'(r)\, m r^2 \dot{\hat{r}}$

Kepler $V'(r) = \frac{\alpha}{r^2}$

$\Rightarrow \frac{d}{dt}\underbrace{(\vec{p} \times \vec{L} - m\alpha\hat{r})}_{\equiv \vec{A}} = 0$



at $r = r_{min}$ $\quad \vec{p} \times \vec{L} \parallel \hat{r}$

$\vec{A}$ is conserved $\Rightarrow$ $\vec{A}$ in direction of long axis

b) algebraic solution

$(\vec{r}\cdot\vec{A}) = \vec{r}\,(-\vec{p}(\vec{p}\cdot\vec{r}) + \vec{r}(\vec{p}\cdot\vec{p})) - k m r$

$= -(\vec{r}\cdot\vec{p})^2 + r^2 p^2 - k m r$

$= r^2 p^2 \sin^2\theta_{rp} - k m r = |\vec{L}|^2 - k m r$

$\vec{r}\cdot\vec{A} = r|\vec{A}|\cos\theta$

$\Rightarrow r = \frac{|\vec{L}|^2}{A\cos\theta + mk}$