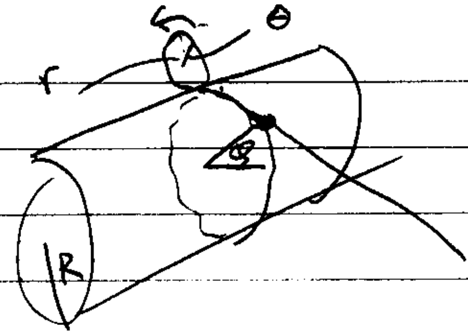
HW 3

2) 

constraints

$$r(d\theta - d\varphi) = R\,d\varphi, \quad R = \text{const.}$$

$$T = \tfrac{1}{2}m\dot{R}^2 + \tfrac{1}{2}m r^2\dot{\theta}^2 + \tfrac{1}{2}m(r+R)^2\dot{\varphi}^2$$

$$U = mg(R+r)\sin\varphi$$

EL eqs. θ ; $mr^2\ddot{\theta} - \lambda r = 0$

φ ; $m(r+R)^2\ddot{\varphi} + \lambda(R+r) + mg(R+r)\cos\varphi = 0$

R ; $m\ddot{R} - m(r+R)\dot{\varphi}^2 + mg\sin\varphi - \lambda_R = 0$

at the point the hoop leaves the cylinder, the generalized force $\lambda_R = 0$

$R = \text{constant} \Rightarrow \ddot{R} = 0$

$\Rightarrow \lambda_R = mg\sin\varphi - m(r+R)\dot{\varphi}^2$

need to express $\dot{\varphi}$ in φ ;

$$(r+R)\lambda = (r+R)m\,\ddot{\theta} = +m\ddot{\varphi}(r+R)^2$$

(small r)

constraint $r\,d\theta = (r+R)\,d\varphi$

$\Rightarrow r\ddot{\theta} = (r+R)\ddot{\varphi}$

$\Rightarrow m(r+R)^2\ddot{\varphi} + m\ddot{\varphi}(r+R)^2 + mg(R+r)\cos\varphi = 0$

$\Rightarrow \frac{d}{dt}\left(m(r+R)\frac{\dot{\varphi}^2}{2} + m(r+R)\frac{\dot{\varphi}^2}{2} + mg\sin\varphi\right) = 0$

$\Rightarrow m(r+R)\dot{\varphi}^2 + mg\sin\varphi = \text{constant}$

$\varphi = \frac{\pi}{2}$ then $\dot{\varphi} = 0 \Rightarrow \text{constant} = mg$

$$\Rightarrow \lambda_R = mg\sin\varphi - m(r+R)\frac{mg(1-\sin\varphi)}{m(R+r)}$$

$= 2mg\sin\varphi - mg \Rightarrow \sin\varphi = \frac{1}{2}$ for $\lambda_R = 0$

$\Rightarrow \varphi = 30°$