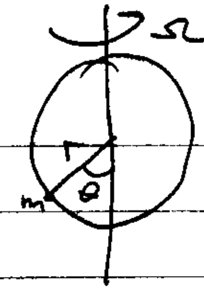
3) $T = \frac{1}{2} m r^2 \sin^2\theta\, \Omega + \frac{1}{2} m r^2 \dot{\theta}^2$



$V = -m g r \cos\theta$

a) $L = \frac{1}{2} m r^2 \dot{\theta}^2 - m g r \cos\theta + \lambda(\dot{\varphi} - \Omega)$
$\quad + \frac{1}{2} m r^2 \sin^2\theta\, \dot{\varphi}^2$

b) φ; $\frac{d}{dt}\left(\lambda + m r^2 \sin^2\theta\, \dot{\varphi}\right) = 0 \Rightarrow$

θ; $\frac{d}{dt}\, m r^2 \dot{\theta} - m r^2 \cos\theta \sin\theta\, \dot{\varphi}^2 + m g r \sin\theta = 0$

equilibrium $\dot{\theta} = 0 \Rightarrow m r^2 (\cos\theta)\dot{\varphi}^2 = m g r$

$$\cos\theta = \frac{g r}{r^2 \Omega^2} = \frac{g}{r\Omega^2}$$

$\theta = \theta_0 + \delta\theta \Rightarrow$

$$m r^2 \delta\ddot{\theta} - m r^2 \cos\theta \sin\theta\, \Omega^2 + m g r \sin\theta = 0$$

$$\cos(\theta_0 + \delta\theta) = \frac{g}{r\Omega^2} - \delta\theta \sin\theta_0$$

$$\sin(\theta_0 + \delta\theta) = \sin\theta_0 + \delta\theta \cos\theta_0$$

$$\Rightarrow m r^2 \delta\ddot{\theta} + m r^2 \delta\theta \sin^2\theta_0\, \Omega^2 + m g r\, \delta\theta \cos\theta_0 = 0$$
$$+ m r^2 \delta\theta(-\cos^2\theta_0)\Omega^2 \quad \left(1 - \frac{g^2}{r^2\Omega^4}\right) \qquad \frac{g}{r\Omega^2}$$

$$\Rightarrow m r^2 \delta\ddot{\theta} + \left(m r^2 \Omega^2 - \frac{m g^2}{\Omega^2} + \frac{m g^2}{\Omega^2}\right)\delta\theta = 0$$
$$-\frac{m g^2}{\Omega^2}$$

$$\Rightarrow m r^2 \delta\ddot{\theta} + \left(m r^2 \Omega^2 - \frac{g^2}{\Omega^2}\right)\delta\theta = 0$$

stable if $m\left(r^2\Omega^2 - \frac{g^2}{\Omega^2}\right) > 0$ or $g < r\Omega^2$