Homework Set # 12 (last set!), due December 4, 2007

1. In this problem we study the standard map numerically. This map, which describes the time evolution of the kicked rotor, is given by

$$J_{n+1} = (\epsilon \sin \phi_n + J_n) \bmod 2\pi, \quad (1)$$
$$\phi_{n+1} = (\phi_n + J_{n+1}) \bmod 2\pi. \quad (2)$$

   Define both ϕ_k and J_k on the interval $[0, 2\pi]$. If the variables are outside the interval subtract a mutiple of 2π. As we did in the lecture we call the map $(\phi_{n+1}, J_{n+1}) \equiv Z_\epsilon(\phi_n, J_n)$.

   - Show a 3d plot of $Z_{\epsilon=0.75}(\phi, J)$.
   - Use as initial conditions 20 equally spaced values of J and a randomly chosen value of ϕ on $[0, 2\pi]$. Iterate the map 500 times for each choice of the initial conditions. Make plots of the iterants for $\epsilon = 0.1,\ 0.5,\ 1.0,\ 2.0,\ 4.0,\ 8.0$. The figures should be similar to the ones shown during the lecture.
   - Take $\epsilon = 1$ and study the evolution of the distance between two initial conditions that are 10^{-8} apart. Compare the evolution for an initial condition in the regular domain (the iterated points are on a curve) and an initial condition in the chaotic domain (the iterated points scatter). Show graphs of the time evolution of the distance between the two trajectories. Also show the iterants of the map in both cases. Take for example 100 iterations.

     Literature: p. 455 of Saletan and Jose, Classical Dynamics or p. 259 of Sussman and Wisdom, Structure and Interpretation of Classical Mechanics.