2a) EL eqs $\frac{d}{dt}\left(m\dot{x}\, e^{\frac{\alpha}{m}t}\right) + kx\, e^{\frac{\alpha}{m}t} = 0$

$\Rightarrow \left(m\ddot{x} + \frac{\alpha}{m} m\dot{x} + kx\right) e^{\frac{\alpha}{m}t} = 0$

2b) $p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}\, e^{\frac{\alpha}{m}t}$

2c) $H = p\dot{x} - L = m\dot{x}^2 e^{\frac{\alpha}{m}t} - \frac{m}{2}\dot{x}^2 e^{\frac{\alpha}{m}t} + \frac{k}{2}x^2 e^{\frac{\alpha}{m}t}$

$= \left(\frac{m}{2}\dot{x}^2 + \frac{k}{2}x^2\right) e^{\frac{\alpha}{m}t}$

$= \frac{p^2}{2m} e^{-\frac{\alpha}{m}t} + \frac{k}{2}x^2 e^{\frac{\alpha}{m}t}$

$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m} e^{-\frac{\alpha}{m}t}$

$\dot{p} = -\frac{\partial H}{\partial x} = -kx\, e^{\frac{\alpha}{m}t}$

2d) $x = e^{\lambda t} \Rightarrow m\lambda^2 + \alpha\lambda + k = 0$

$\Rightarrow \lambda_{1,2} = \frac{-\alpha \pm \sqrt{\alpha^2 - 4km}}{2m}$

general solution $x(t) = A e^{\lambda_1 t} + B e^{\lambda_2 t}$

$A + B = x(0)$

$\dot{p} = -k e^{\frac{\alpha}{m}t}\left(A e^{\lambda_1 t} + B e^{\lambda_2 t}\right)$

$\Rightarrow p = c - \frac{kA}{\lambda_1 + \frac{\alpha}{m}} e^{\left(\frac{\alpha}{m} + \lambda_1\right)t} - \frac{kB}{\lambda_2 + \frac{\alpha}{m}} e^{\left(\frac{\alpha}{m} + \lambda_2\right)t}$

simpler is to use $p = m\dot{x}\, e^{\frac{\alpha}{m}t}$

$\Rightarrow p = m\left(\lambda_1 A e^{\lambda_1 t} + B\lambda_2 e^{\lambda_2 t}\right) e^{\frac{\alpha}{m}t}$

$\Rightarrow p(0) = m\left(A\lambda_1 + B\lambda_2\right)$

$-A + x(0)$

$\frac{p(0)}{m} + m\, x(0)\lambda_2 = A(\lambda_1 - \lambda_2)$