

2)

p, q canonical

$$\dot{q} = -\frac{\partial H}{\partial p} \quad \dot{p} = \frac{\partial H}{\partial q}$$

$$[Q_i, Q_j]_{p, q} = [P_i, P_j]_{p, q} = 0$$

$$[Q_i, P_j]_{q, p} = \delta_{ij}$$

show that $\frac{dQ}{dt} = \frac{\partial H}{\partial p}$ $\frac{dP}{dt} = -\frac{\partial H}{\partial Q}$

$$\begin{aligned} \frac{dQ_i}{dt} &= \frac{\partial Q_i}{\partial q_j} \frac{dq_j}{dt} + \frac{\partial Q_i}{\partial p_j} \frac{dp_j}{dt} \\ &= \frac{\partial Q_i}{\partial q_j} \frac{\partial H}{\partial p_j} - \frac{\partial Q_i}{\partial p_j} \frac{\partial H}{\partial q_j} \\ &= \frac{\partial Q_i}{\partial q_j} \left(\frac{\partial H}{\partial p_e} \frac{\partial p_e}{\partial p_j} + \frac{\partial H}{\partial Q_e} \frac{\partial Q_e}{\partial p_j} \right) \\ &\quad - \frac{\partial Q_i}{\partial p_j} \left(\frac{\partial H}{\partial p_e} \frac{\partial p_e}{\partial q_j} + \frac{\partial H}{\partial Q_e} \frac{\partial Q_e}{\partial q_j} \right) \\ &= \frac{\partial H}{\partial p_e} [p_e, Q_i] + \frac{\partial H}{\partial Q_e} [Q_e, Q_i] \\ &= \frac{\partial H}{\partial p_e} \end{aligned}$$

$$\begin{aligned} \frac{dP_i}{dt} &= \frac{\partial P_i}{\partial q_j} \frac{dq_j}{dt} + \frac{\partial P_i}{\partial p_j} \frac{dp_j}{dt} \\ &= \frac{\partial P_i}{\partial q_j} \frac{\partial H}{\partial p_j} + \frac{\partial P_i}{\partial p_j} (-) \frac{\partial H}{\partial q_j} \\ &= \frac{\partial P_i}{\partial q_j} \left(\frac{\partial H}{\partial Q_e} \frac{\partial Q_e}{\partial p_j} + \frac{\partial H}{\partial p_e} \frac{\partial p_e}{\partial p_j} \right) - \frac{\partial P_i}{\partial p_j} \left(\frac{\partial H}{\partial Q_e} \frac{\partial Q_e}{\partial q_j} + \frac{\partial H}{\partial p_e} \frac{\partial p_e}{\partial q_j} \right) \\ &= \frac{\partial H}{\partial Q_e} [Q_e, P_i] = -\frac{\partial H}{\partial Q_i} \end{aligned}$$