\[ p, q \quad \text{canonical} \]
\[ \dot{p} = -\frac{\partial H}{\partial q}, \quad \dot{q} = \frac{\partial H}{\partial p} \]
\[ [Q_i, Q_j]_{p, q} = [P_i, P_j]_{p, q} = 0 \]
\[ [Q_i, P_j]_{q, p} = \delta_{ij} \]

Show that
\[ \frac{dQ_i}{dt} = \frac{\partial H}{\partial P_i} \quad \frac{dP_i}{dt} = -\frac{\partial H}{\partial Q_i} \]

\[ \frac{dQ_i}{dt} = \frac{\partial Q_i}{\partial q_j} \frac{dq_j}{dt} + \frac{\partial Q_i}{\partial p_j} \frac{dp_j}{dt} \]
\[ = \frac{\partial Q_i}{\partial q_j} \frac{\partial H}{\partial q_j} - \frac{\partial Q_i}{\partial p_j} \frac{\partial H}{\partial p_j} \]
\[ = \frac{\partial Q_i}{\partial q_j} \left( \frac{\partial H}{\partial q_j} \frac{\partial P_j}{\partial q_j} + \frac{\partial H}{\partial q_j} \frac{\partial Q_j}{\partial q_j} \right) \]
\[ \quad - \frac{\partial Q_i}{\partial p_j} \left( \frac{\partial H}{\partial p_j} \frac{\partial P_j}{\partial q_j} + \frac{\partial H}{\partial p_j} \frac{\partial Q_j}{\partial q_j} \right) \]
\[ = \frac{\partial H}{\partial q_j} [-Q_j, Q_i] + \frac{\partial H}{\partial Q_j} [Q_j, Q_i] \]
\[ = \frac{\partial H}{\partial q_j} \]

\[ \frac{dP_i}{dt} = \frac{\partial P_i}{\partial q_j} \frac{dq_j}{dt} + \frac{\partial P_i}{\partial p_j} \frac{dp_j}{dt} \]
\[ = \frac{\partial P_i}{\partial q_j} \frac{\partial H}{\partial q_j} + \frac{\partial P_i}{\partial p_j} \left( -\frac{\partial H}{\partial q_j} \right) \]
\[ = \frac{\partial P_i}{\partial q_j} \left( \frac{\partial H}{\partial q_j} \frac{\partial Q_j}{\partial q_j} + \frac{\partial H}{\partial q_j} \frac{\partial Q_j}{\partial q_j} \right) - \frac{\partial P_i}{\partial p_j} \left( \frac{\partial H}{\partial p_j} \frac{\partial Q_j}{\partial q_j} + \frac{\partial H}{\partial p_j} \frac{\partial Q_j}{\partial q_j} \right) \]
\[ = \frac{\partial H}{\partial Q_j} [Q_j, P_i] = -\frac{\partial H}{\partial Q_i} \]