

$$2g(z-z_0) = 2g(t-t_0) \sqrt{\frac{2E}{m} - 2gz_0} - g^2(t-t_0)^2$$

$$z = z_0 + g(t-t_0) \sqrt{\frac{2E}{m} - 2gz_0} - \frac{1}{2}g^2(t-t_0)^2$$

$$E = \frac{1}{2}m v_0^2 + m g z_0$$

$$\Rightarrow z = z_0 + v_0(t-t_0) - \frac{1}{2}g(t-t_0)^2$$