1a) \[ L = \frac{1}{2} m k^2 \dot{\theta}^2 \]

1b) \[ \frac{d}{dt} \frac{dL}{d\dot{\theta}} = 0 = \frac{d}{dt} m k^2 \dot{\theta} = 0 \] i.e. angular momentum is conserved

2a) \[ \dot{\dot{\theta}} = 0 \quad L_2: \quad \frac{d}{dt} (\dot{\theta} + q) - \ddot{q} = 0 \]

\[ \Rightarrow \frac{d}{dt} \dot{q} = 0 \]

2b) \[ -L_1 + L_2 = q \ddot{q} = \frac{d}{dt} \left( \frac{1}{2} q^2 \right) \]

Eqs. of motion are the same if Lagrangian differs by a total time derivative.