At the opposite, if no relation of the type (III-2) holds, the orbit never closes, but densely covers the torus after infinite time; such a spiralling orbit is "ergodic" on the torus (see footnote p67) -called "irrational", or "non-resonant" torus-, and exhibits a strong regularity. Intermediate cases where \( p \) independent relations \( (0 < p < N-1) \) like (III-2) hold can also occur; orbits then lie on \((N-p)\)-dimensional manifolds of \( T^N \).

It should be noticed that in the generic integrable case (see below the harmonic oscillator and the Kepler motion as exceptions), the frequencies depend on the actions (which define the invariant tori), i.e. on the set of initial conditions fixing the values of the \( N \) first integrals of the motion. Consequently for a generic integrable system, there exist simultaneously -corresponding to different sets of initial conditions- non-resonant tori, covered by ergodic trajectories and resonant tori; measure theoretic arguments show that for such systems, the irrational tori form a set which is almost everywhere dense [Ar-76]. In other words, almost all the tori of a generic integrable system are irrational, in the same way as almost all real numbers are irrational.

**Examples of regular systems**

i) for \( N=1 \), we already saw that all conservative systems are regular (therefore periodic).

ii) for \( N=3 \), all systems submitted to a central field force \( V(r) \) are regular: \( L^2 \) and \( L_Z \) (orbital moment) are conserved, together with \( H \).

Two particular systems of this kind are of special interest:

- for the Kepler motion \( (V(r) = -\frac{\mu}{r}) \):

  \[
  H = \frac{P_r^2}{2m} + \frac{P_\theta^2}{2mr^2} + \frac{P_r^2}{2mr^2 \sin^2 \theta} - \frac{\mu}{r}.
  \]

  The Hamiltonian is well known to write, in terms of the action variables \( I_\xi = \oint P_\xi \, dq_\xi \) (each integral is over the period corresponding to the \( q_\xi \)) as:

  \[
  \tilde{H} = \frac{-2\pi^2 m k^2}{(I_r + I_\theta + I_\phi)^2}.
  \]

- for the isotropic harmonic oscillator \( (V(r) = \omega^2 \frac{P^2}{2}) \), the Hamiltonian takes the form

  \[
  \tilde{H} = \omega \left( I_1 + I_2 + I_3 \right).
  \]

In both cases, for all values of the actions, i.e. for all initial conditions, the three frequencies coincide; the orbits are closed, with period \( \tau = \frac{2\pi}{\omega} \). This means that for these two particular systems, all tori are rational (no orbit covering densely a torus). To understand why such a situation is an exception, one has to remember Bertrand's theorem [Be-1873]: Consider the motion of a point-mass under the action of a spherically