• The asymptotic relative frequency $P_k$ of $k$ among the partial quotients $a_1(x), a_2(x), \ldots, a_n(x), \ldots$, is known to be

$$P_k = \frac{1}{\ln 2} \ln \left( \frac{(k+1)^2}{k(k+1)} \right)$$

for almost all $x$.

• $\lim_{n \to \infty} \sqrt[n]{a_1(x)} \cdots a_n(x) = \prod_{k=1}^{\infty} \left( 1 + \frac{1}{k^2 + 2k} \right)^{\ln k / \ln 2}$ \textit{a.e.} \quad ("a.e." stands for "almost everywhere")

• $\lim_{n \to \infty} \frac{1}{n} \ln \left| \frac{P_n(x)}{q_n(x)} \right| = -\frac{\pi^2}{6 \ln 2} = -h_{\text{ks}}(r)$ \textit{a.e.}

where $\frac{P_n(x)}{q_n(x)}$ is the $n^{\text{th}}$-order convergent of $x$ (or rational approximation to $x$):

$$\frac{P_n(x)}{q_n(x)} = \frac{1}{a_1(x) + \frac{1}{a_2(x) + \cdots}} \quad + \frac{1}{a_n(x)}$$

($P_n(x)$ and $q_n(x)$ are prime to each other)

• $\lim_{n \to \infty} \frac{1}{n} \ln q_n(x) = \frac{\pi^2}{12 \ln 2}$ \textit{a.e.}

Notice that all the properties derived from ergodicity hold on $X = [0,1[$, \textit{except for a set of numbers of zero measure}. In particular, rational numbers are trivially excluded from $X$ for the applicability of these results, since they have only a finite number of non-zero partial quotients $a_k$. Also excluded are quadratic irrational numbers (i.e. irrational roots of a quadratic equation with integer coefficients), whose sequence of partial quotients $\{a_k(x)\}$ becomes periodic for sufficiently large values of $k$: for any quadratic irrational, there exist two integers $k_0$ and $p$ such that, for every $k > k_0$, $a_k = a_{k+p}$. (Evidently, Eq.(III-17) cannot hold for such numbers).