The Lagrangian for a free particle

\[ L = \frac{1}{2} m \dot{\mathbf{x}}^2 - V(\mathbf{x}) \]

The second term on the right side of this expression is the kinetic energy of the particle.

Expanding this expression, we obtain

\[ (\dot{x} + x) \frac{\partial^2 \mathbf{x}}{\partial t^2} + (\ddot{x} + x) \frac{\partial^2 \mathbf{x}}{\partial t^2} = (\dot{x} + x) \frac{\partial^2 \mathbf{x}}{\partial t^2} \]

We have

The Lagrangian for motion of the particle

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The equation of motion is obtained by minimizing this functional with respect to \( \mathbf{x}(t) \). If we neglect the second derivative terms, we obtain the equation of motion. If we neglect the first derivative terms, we obtain the equation of motion for a free particle.

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