Midterm II, PHY306, April 26 2006, 9.50-11.10

1.

a) Show that the efficiency for a Carnot cycle is given by $(\tau_h - \tau_l)/\tau_h$.

b) What is the Fermi energy? (answer in one line)

c) Show that for a liquid to gas transition at constant temperature and pressure $\mu_g = \mu_l$. (Hint: use that the Gibbs free energy is minimized.)

d) Show that the grand canonical partition function of fermions in energy levels ϵ_k is given by $\mathcal{Z} = \prod_k (1 + \lambda e^{-\epsilon_k/\tau})$.

2. Consider the following reversible cycle of an ideal gas. Express all your answers in terms of V_l, V_h, p_l, p_h and the number of particles N. (Hint: use that $pV = N\tau$ and that the internal energy $U = \frac{3}{2}N\tau$.)

a) What is the total work done in one cycle.

b) Calculate the heat added going from 1 to 2.

c) Calculate the heat added going from 2 to 3.

d) Calculate the heat added going from 3 to 4.

e) Calculate the heat added going from 4 to 1.

f) Check that the total heat added is the work done in one cycle.

3.

a) Show that the average pressure in a system in thermal contact with a heat reservoir is given by

$$p = \frac{1}{Z} \sum_s (\partial \epsilon_s / \partial V)_N \exp(-\epsilon_s/\tau), \quad (1)$$

where the sum is over all states of the system.

b) Show that for a gas of free particles

$$(\partial \epsilon_s / \partial V)_N = -\frac{2}{3}\frac{\epsilon_s}{V} \tag{2}$$

(Hint: use that $\epsilon_s = \frac{\hbar^2}{2M}\frac{\pi^2}{V^{2/3}}(n_1^2 + n_2^2 + n_3^2)$)

c) Show that for a gas of free nonrelativistic particles

$$p = \frac{2}{3}\frac{U}{V}, \tag{3}$$

where U is the internal energy of the system.