1.

a) Show that the entropy always increases if two subsystems are brought in thermal contact.

b) Show that for an ideal gas

\[
\frac{\partial \sigma}{\partial V} \bigg|_\tau = \frac{\partial p}{\partial \tau} \bigg|_V. \tag{1}
\]

c) Show that \( C_p = \frac{5}{2} N \) for an ideal gas.

d) Show that for an ideal gas, \( Z_N = (Z_1)^N / N! \).

2. Use the partition function to find an exact expression for the magnetization \( M \) and the susceptibility \( \chi \equiv dM/dB \) as a function of temperature and magnetic field for the model system of magnetic moments in a magnetic field. The result for the magnetization is \( M = nm \tanh(mB/\tau) \), where \( n \) is the particle concentration and \( m \) the magnetic moment of the particles.

3. Consider a gas of bosons that can either be in a state with energy \( \epsilon_0 = 0 \) or in a state with energy \( \epsilon_1 = \epsilon \).

a) Calculate the grand canonical partition function as a function of the chemical potential and temperature.

b) Derive an expression for the average number of particles \( \langle N \rangle \).

c) Find the low temperature limit of \( \langle N \rangle \) for \( \mu \to \epsilon \).