Tracing Gluon Exchange from Short Distances to Long

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on the happy occasion of Ed Shuryak’s 60th
• Motivations

• The logic of perturbative QCD: energy flow & jets

• Jet measures and the perturbative/nonperturbative transition

• Finding a basic exchange

• Extension to partonic scattering
MOTIVATIONS

• QCD: what’s all the fuss still about?

• Simple laws to complex behavior is a central theme of twenty-first century physics.

• I’ll discuss one aspect: how the short-distance physics of jets evolves to long distances, with the hint of a more general formulation.
The logic behind perturbative QCD

The sorrows of QCD perturbation theory:

1. Confinement

\[ \int e^{-iq \cdot x} \langle 0 | T[\phi_a(x) \ldots] | 0 \rangle \]

has no \( q^2 = m^2 \) pole for any field (particle) \( \phi_a \) that transforms nontrivially under color (confinement)

2. The pole at \( p^2 = m^2_\pi \)

\[ \int e^{-iq \cdot x} \langle 0 | T[\pi(x) \ldots] | 0 \rangle \]

is not accessible to perturbation theory (\( \chi_{SB} \) etc., etc.)
• But some cross sections can be expanded in $\alpha_s$:

$$Q^2 \hat{\sigma}(Q^2) = \sum_n c_n(1) \alpha_s^n(Q) + \mathcal{O}(1/Q^p)$$

• What are we really calculating?
  PT for color singlet currents via the optical theorem:

• Necessary combination: no observed color charges; insensitivity to long distances: “infrared safety”.

• e.g., $e^+e^-$ total cross section. “One scale, $Q$.”

$$Q^2 \hat{\sigma}(Q^2) \propto \text{Im} \int d^4x e^{-iQ \cdot x} \langle 0 | T[J_{\text{em}}(x)J_{\text{em}}(0)] | 0 \rangle$$
– Another class of color singlet matrix elements:


. . . Bauer, Fleming, Lee, GS (08) Hofmann & Maldacena (08)

\[
\lim_{R \to \infty} R^2 \int dx_0 \int d\hat{n} f(\hat{n}) e^{-i q \cdot y} \langle 0 | J(0) T[\hat{n}_i T_{0i}(x_0, R\hat{n}) J(y)] | 0 \rangle
\]

With \( T_{0i} \) the energy momentum tensor

– These are what we really calculate.

If the “weight” \( f(\hat{n}) \) introduces no new dimensional scale, and all \( d^k f / d\hat{n}^k \) bounded, then individual final states have IR divergences, but these cancel in sum over collinear splitting/merging & soft parton emission because they respect energy flow.
A jet is a set of particles that is the product of energy-flow-respecting interactions.

A jet’s internal structure encodes its history. How is energy spread out by radiation/hadronization? How are particles distributed in energy?
Factorization and Resummation

- **Generalize infrared safety:**

  \[ Q^2 \sigma_{\text{phys}}(Q, m) = \omega_{SD}(Q/\mu, \alpha_s(\mu)) \otimes f_{LD}(\mu, m) + \mathcal{O}(1/Q^p) \]

- \( \mu \) = factorization scale; \( m \) = IR scale (\( m \) may be perturbative)

- **New physics in** \( \omega_{SD} \); \( f_{LD} \) “universal”

- **Central to hadron collider physics.**

- Factorization: products of probabilities. This is a paradox for a quantum theory.
Factorization proofs:

- (1) $\omega_{SD}$ incoherent with long-distance dynamics
  The operator product expansion.

- (2) Mutual incoherence when $v_{\text{rel}} = c$:
  Jet-jet factorization

- (3) Wide-angle soft radiation sees only total color flow:
  jet-soft factorization

- (4) Dimensionless coupling and renormalizability
• Whenever there is factorization, there is evolution

\[ 0 = \mu \frac{d}{d\mu} \ln \sigma_{\text{phys}}(Q, m) \]

\[ \mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu} \]

**PDF** \( f \) or **Fragmentation** \( D \)

• Wherever there is evolution there is resummation

\[ \ln \sigma_{\text{phys}}(Q, m) = \exp \left\{ \int_{Q}^{\mu} \frac{d\mu'}{\mu'} P(\alpha_s(\mu')) \right\} \]
Application: Jet Measures of the P/NP Transition

- **Illustrative example:** Flexible event shapes
  
  (C.F. Berger, Kúcs, GS (2003), Berger, Magnea (2004))

\[
\tau_a = \frac{1}{Q} \sum_{i \in N} p_{Ti} e^{-(1-a)|\eta_i|} = \frac{1}{Q} \sum_{i \in N} E_i (\sin \theta_i)^a (1 - |\cos \theta_i|)^{1-a}
\]

- \(p_{Ti}, \eta_i, \theta_i\), rel. to axis with min. \(\tau_0\) (jet-by-jet).

- Thrust: \(a = 0\); broadening: \(a = 1\); inclusive limit \(a \to \infty\).

- For multijet final states, define \(\eta_i\) relative to closest jet.
– **Factorization:** \( \sigma \) a convolution in contributions of each jet with an “interjet” soft radiation function

\[
\sigma(\tau_a, Q, a) = H(Q) \int dt_s \prod_{\text{jets } i} \int dt_i S(t_s) \prod_i J_i(t_i, p_{J_i}) 
\times \delta\left(\sum_i t_i + t_s - \tau_a\right)
\]

– Thus, general resummed cross section can be written as an inverse Laplace transform

\[
\sigma(\tau_a, Q, a) = H(Q) \int_{-i\infty}^{i\infty} d\nu e^{\nu \tau_a} S(\nu) \prod_i J_i(\nu, p_{J_i})
\]

where \( f(\nu) \equiv \int_0^\infty dt \, e^{-\nu t} f(t) \).

– Three-way factorization \( \Rightarrow \) CO/IR (Sudakov) resummation
• The jet in Laplace transform space

\[ J_i(\nu, p_J) = \int_0^1 d\tau e^{-\nu \tau J_i} J_i(\tau J_i, p J_i) = e^{\frac{1}{2} E(\nu, Q, a)} \]

\[ E(\nu, Q, a) = 2 \int_0^1 du \left[ \int_{u^2 Q^2}^{u Q^2} \frac{dp_T^2}{p_T^2} A(\alpha_s(p_T)) \left( e^{-u^{1-a} \nu (p_T/Q)^a} - 1 \right) \right] \]

\[ + \frac{1}{2} B(\alpha_s(\sqrt{uQ})) \left( e^{-u(\nu/2)^2/(2-a)} - 1 \right) \]

• Two logarithmic enhancements, \((\text{energy}, p_T)\) but

• the expansion in \(\alpha_s(Q)\) finite at all orders.
Emergent nonperturbative effects

- How to interpret expressions like

\[ E(\nu, Q, a) = 2 \int_0^1 \frac{du}{u} \left[ \int \frac{dp_T^2}{u^2 Q^2} A(\alpha_s(p_T)) \left( e^{-u^{1-a} \nu (p_T/Q)^a} - 1 \right) \right. \]

\[ + \frac{1}{2} B(\alpha_s(\sqrt{u} Q)) \left( e^{-u(\nu/2)^{2/(2-a)}} - 1 \right) \]

Nonperturbative scales for \( u, p_T \to 0 \)
• Organizing the NP corrections: shape functions.

- $p_T > \kappa \equiv E_{PT}$,
- $p_T < \kappa$: expand in powers of $1/Q$

$$E(\nu, Q, a) = E_{PT}(\nu, Q, \kappa, a)$$

$$+ \frac{2}{1 - a} \sum_{n=1}^{\infty} \frac{1}{n n!} \left(-\frac{\nu}{Q}\right)^n \int_0^{\kappa^2} \frac{dp_T^2}{p_T^2} p_T^n A(\alpha_s(p_T)) + \ldots$$

$$\equiv E_{PT}(\nu, Q, \kappa, a) + \ln \tilde{f}_{a, NP} \left(\frac{\nu}{Q}, \kappa\right)$$

Get a factorized “shape function” $f_{NP}$ (additive in $E$).
• PT & $f_{NP}$ in Laplace moments $\rightarrow$ convolution:

$$\sigma(\tau_a, Q) = \int d\xi \sigma_{PT}(\tau_a - \xi, Q) f_{a, NP}(\xi)$$

• $e^+e^-$: fit at $Q = M_Z \Rightarrow$ predictions for all $Q$, any (quark) jet.

• Portable to jets in hadronic collisions.

• And sensitive to gluon/quark origin of the jet
Shape function phenomenology for thrust at LEP.

(Korchemsky, GS, Belitsky; Gardi Rathsman, Magnea (1998 . . . ))
Finding a Basic Exchange

• Find the anomalous dimension $A$ among gluons

• A typical diagram for final-state recoil-less colored particles, a.k.a. “Wilson lines”. (*Gluons only see their velocities, $\beta$.*)
Webs and exponentiation in jet cross sections

(GS; Gatheral, Frenkel and Taylor, 1981)

\[
\frac{d\sigma}{d\tau_a} = \sum_{n=0}^{\infty} \frac{1}{n!} \int de \delta(\tau_a - \sum e_i) \prod_{i=1}^{n} E(e_i)
\]

\[
E(e) = \sum_{\text{states}} W_n(e) = \sum_{M} C(M_n) M_n^2(e)
\]

The $M_n^2$ are momentum integrals.
• The $C(\mathcal{M}_n)$ are modified color factors for $\mathcal{M}_n$s.
  Examples at $\alpha_s^2$:

  \[ \text{all } C(M) = C_F C_A \]

• Can’t be disconnected by cutting 2 “eikonal” lines: “webs”.

\[ \text{Diagram showing graphs with no disconnection by cutting 2 eikonal lines.} \]
• The webs determine exponentiation under Laplace transforms:

\[ \tilde{\sigma}_e(\nu) \equiv \int de \ e^{-\nu \tau_a} \frac{d\sigma}{d\tau_a} = \exp \left[ \int de \ e^{-\nu e} E(e) \right] \]

• Double logarithmic behavior is encoded in the construction of the webs \( \mathcal{W} \). Subdivergences cancel.

• Each web gives a single collinear and infrared logarithm just like a single gluon.

• In a theory with a fixed coupling (SYM . . . ) a web acts exactly like a single gluon.
• For some cross sections, this gives a very specific template \( k_0 = \text{energy of radiated gluons} \):

• Boost invariance:

\[
\lim_{\nu \to \infty} \ln \hat{\sigma}^{(eik)}(\nu, Q) = \sum_j \int dPS_j \theta(Q^2 - k^2) |M^{(PT)}|^2 \ e^{-\nu k_0 / Q} \\
= \int_0^{Q^2} \frac{\rho(\alpha_s(u, \varepsilon))}{u^2} \left[ K_0 \left( \frac{2\nu u}{Q} \right) + \ln \frac{u}{Q} \right] + \ln \nu \int_0^{Q^2} \frac{du^2}{u^2} A(\alpha_s(u, \varepsilon))
\]

• The web as anomalous dimension: \( \rho(\alpha_s(u, \varepsilon)) = A(\alpha_s(u, \varepsilon)) + \frac{\partial D}{\partial \ln \mu^2} \)

• \( A \): as above; \( D(\alpha_s, 0) \) seen at NNLO.

• \( \rho, A \), gluon exchange now have nonperturbative interpretation.
Extension to Partonic Scattering/Production

Evidence beyond pair production.

- To generalize,


- organize multi-parton amplitudes by color exchange:

\[ f : f_A(p_A, r_A) + f_B(p_B, r_B) \rightarrow (\text{or } +) f_1(p_1, r_1) + f_2(p_2, r_2) \]

\[ \mathcal{M}^{[f]}_{\{r_i\}} \left( p_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = \mathcal{M}^{[f]}_L \left( p_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) (c_L)_{\{r_i\}} \]

- Need to control logs & poles in \( \epsilon = 2 - d/2 \).
Example: $q\bar{q}$ tensors $(c_L)\{r_i\}$:

\[
\begin{array}{ccc}
\begin{array}{ccc}
1 & 3 & 1 \\
& \uparrow & \\
& & \\
2 & 4 & 2
\end{array}
& \equiv &
\begin{array}{ccc}
\begin{array}{ccc}
1 & 3 & 1 \\
& \downarrow & \\
& & \\
2 & 4 & 2
\end{array}
\end{array}
\]

Jet/soft factorization at amplitude level:

(Sen (1983))

\[
\mathcal{M}_L^{[f]} \left( p_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = \prod_{i=A,B,1,2} J_i^{[\text{virt}]} \left( \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right)
\]

\[
\times S_{LI}^{[f]} \left( p_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) h_I^{[f]} \left( \varphi_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right)
\]
• Source of double logs and poles in dimensional reg.:

`Leading Regions`:

\[ p_1 \quad \quad \quad \quad p_3 \]
\[ p_2 \quad \quad \quad \quad p_4 \]

\[ = \]

`soft`

\[ p_1 \quad \quad \quad \quad p_3 \]
\[ p_2 \quad \quad \quad \quad p_4 \]

`hard`

`jets`

• The same cast of characters as for jets.
• Same separation:

Factorization of soft gluons:

\[ \sum I \]

• \( \varepsilon = 2 - d/2 \) the "IR scale".
- Jet functions $J_i$ are same for all processes.

- Soft function labelled by color exchange (singlet, octet . . . )

- Verified in supersymmetric Yang-Mills theories to 4 loops.  
  (Bern, Czakon, Dixon, Kosower & Smirnov (2006))

- Same singularity structure in the strong-coupling limit for large $N_c$ super-YM. 
  (Alday and Maldacena (2007))
– Dimensionally-regularized $S$

$$S^{[f]} \left( \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right)$$

$$= P \exp \left[ -\frac{1}{2} \int_0^{-Q^2} \frac{d\tilde{\mu}^2}{\tilde{\mu}^2} \Gamma^{[f]} \left( \tilde{\alpha}_s \left( \frac{\mu^2}{\tilde{\mu}^2}, \alpha_s(\mu^2), \epsilon \right) \right) \right]$$

$\Gamma^{[f]}$: anomalous dimension; color mixing

- All the effects of multiple partons are in this matrix.

- All expectations were that soft gluons would begin to couple to three partons “at a time” starting at NNLO, but . . .
• Result for all massless $2 \rightarrow n$ processes

(Aybat, Dixon, GS (2006))

$$\Gamma_S = \frac{\alpha_s}{\pi} \left( 1 + \frac{\alpha_s}{\pi} K \right) \Gamma_{S'}^{(1)} + \cdots$$

$$\Gamma^{(2)} = (K/2)\Gamma^{(1)} \text{ with same constant } K \text{ as in the } A \text{ above:}$$

$$A(\alpha_s) = \frac{\alpha_s}{\pi} \left( 1 + \frac{\alpha_s}{2\pi} K \right) A^{(1)}$$
• How it happens:

The diagrams with 3g vertices vanish!
The full two-loop single-pole terms $\times \ LO$ are simply

$$\left[ \sum_{i \in f} \frac{E^{[i]}_1 (2)}{\varepsilon} + \frac{1}{4\varepsilon} \Gamma^{[f]}_S (2) \right] \times \ LO$$

$E^{[i]}_1 (2)$ is 2 loop single pole in eikonal form factor

(Ravindran, Smith, van Neerven (2005))

“Web exchange” is retained to NNLO for multiparton processes.

An unexpected simplicity in the IR structure of QCD.

A SIGN OF THE “REAL INFRARED GLUON”?
HAPPY BIRTHDAY, EDWARD!